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by

H.F. Tiersten

Office of Naval Research
Contract N00014-76-C-0368
Project NR 318-009
Report No. 33
Final Report

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER No. 33	2. GOVT ACCESSION NO. AD-A100 144
3. RECIPIENT'S CATALOG NUMBER Final rept.	
4. TITLE (and Subtitle) (6 ELECTROMAGNETISM IN INTERACTION WITH DEFORMABLE SOLIDS.	5. ~~TYPE OF REPORT & PERIOD COVERED~~ Feb[redacted]69-Oct[redacted]80, Final Report
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) H.F. Tiersten 14) 33 15	8. CONTRACT OR GRANT NUMBER(s) N00014-76-C-0368
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Mechanical Engineering, Aeronautical Engineering & Mechanics Rensselaer Polytechnic Institute Troy, New York 12181	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NR318-009 (12) 47
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research Physics Branch 11	12. REPORT DATE May [redacted]81
	13. NUMBER OF PAGES 44
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report) Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Distribution of this document is unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Interactions	Continuum Mechanics	Piezoelectricity	~~Wave Propagation~~ Plates
Deformable Solids	Nonlinear	Elasticity	Rods
Electromagnetism	Insulators	Electroelasticity	Thin Films
Solid Continua	Conductors	Magnetoelasticity	Surface Waves
Continuum Physics	Semiconductors	Vibrations	Composite Materials

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This is the final report on research begun ~~under~~ Contract No. N00014-67-A-0117-0007 and continued under Contract No. N00014-76-C-0368. The research effort encompasses three subject areas consisting of the following:

1. Electromagnetism in interaction with conducting and non-conducting polarizable and magnetizable deformable solid continua.

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S/N 0102-LF-014-6601

Unclassified
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

2. The vibrations of piezoelectric plates and rods.
3. Elastic surface waves guided by thin films.

ELECTROMAGNETISM IN INTERACTION WITH DEFORMABLE SOLIDS

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ABSTRACT

This is the final report on research begun under Contract No. N000?4-67-A-0117-0007 and continued under Contract No. N00014-76-C-0368. The research effort encompasses three subject areas consisting of the following:

1. Electromagnetism in interaction with conducting and nonconducting polarizable and magnetizable deformable solid continua.
2. The vibrations of piezoelectric plates and rods.
3. Elastic surface waves guided by thin films.

1. Introduction

This is the final report on research begun under Contract No. N00014-67-A-0117-0007 and continued under Contract No. N00014-76-C-0368 with the Office of Naval Research. The research effort encompasses three distinct subject areas consisting of the following:

1. Electromagnetism in interaction with conducting and nonconducting polarizable and magnetizable deformable solid continua.
2. The vibrations of piezoelectric plates and rods
3. Elastic surface waves guided by thin films.

In subject area 1 the description of the interaction of the quasi-static electric field with a finitely deformable, polarizable, nonconducting continuum has been obtained[1]. The resulting equations are applicable in the description of the dynamic nonlinear behavior of high coupling piezoelectric materials such as lithium niobate and the linear behavior of such materials under static or slowly varying biasing fields. In particular, these equations are required for a consistent description of both bulk and surface wave parametric effects in such materials in terms of the fundamental material parameters. The equations in the small, but nonlinear (quadratic), field variables for small fields superposed on a static bias have been obtained[2] from the aforementioned general nonlinear interaction equations[1]. These more tractable equations have been applied to the problem of the thickness vibrations of high coupling piezoelectric plates subject to static biasing electric fields[2] and to the problems of second harmonic generation and parametric excitation of surface waves in isotropic elastic and anisotropic piezoelectric solids[3]. The solution to the nonlinear surface wave problem for anisotropic piezoelectric materials is essential for the analytic determination of practical orientations, if indeed there be any, for surface wave parametric interactions in high coupling materials such as lithium niobate. The linear limit of the equations for small fields superposed on a bias has been applied[4] in a direct calculation of the influence of a flexural biasing state on the velocity of surface waves in piezoelectric solids. The results of this investigation have been useful in the interpretation of measurements on pressure sensing acoustic surface wave transducers[5].

A perturbation formulation of the linear electroelastic equations for small fields superposed on a bias has been obtained[6]. The bias may be either electrical or mechanical or both. This formulation provides a relatively simple extremely accurate description of the influence of biasing electric fields, stresses, and strains on such things as the velocity of surface waves and the resonant frequencies of vibrating piezoelectric structures once the bias is known. The perturbation formulation has been applied[4] to the aforementioned problem concerning the influence of a flexural biasing state on the velocity of piezoelectric surface waves and it has been shown that the perturbation procedure has a significant advantage over a direct calculation since it can treat spatially varying biasing states, which cannot be treated by means of a direct calculation. The spatial variation of the biasing state has been shown to have an appreciable influence well within the practical range of pressure transducers. In addition, the perturbation formulation has been applied in the determination of the temperature dependence of the resonant frequency of electroded quartz plates vibrating in thickness-modes[7]. In order to calculate the actual frequency change for doubly-rotated quartz plates, the first temperature derivatives of the fundamental elastic constants of quartz had to be determined[8] from the original data from which the existing temperature derivatives of the effective constants were obtained[9]. The newly defined temperature derivatives[8] are required because the measured data has to be related to the proper linear equations for small fields superposed on a bias, which include the third order elastic constants, in order to be used in the calculation of frequency changes resulting from thermal stresses and deformations. In order to find the temperature induced biasing state a system of approximate plate equations for the determination of thermal stresses in electroded piezoelectric plates was obtained[7]. The resulting approximate equations simplify the treatment of many thermal stress problems considerably and have been applied in the determination of thermal stresses in electroded quartz plates. In addition to its use in the calculation of the frequency change in electroded doubly-rotated quartz thickness-mode resonators[7], the calculated thermoelastic deformation and the first temperature derivatives of the fundamental elastic constants of quartz[8] have been employed along with the linear modal solutions[10,11] in the equation for the perturbation in eigenfrequency[6] to obtain the time-independent change in resonant frequency with temperature for both contoured AT-cut quartz crystal resonators[12] and AT-cut quartz trapped energy resonators[13]. The results obtained are related to the dependence of the apparent shift in angle of the zero temperature cut on either the contouring[12] or the electrode geometry[13]. Furthermore, time-dependent changes in resonant frequency resulting from transient inhomogeneous temperature excursions have been obtained in the case of doubly-rotated quartz thickness-mode resonators[14] in essentially the same way, and it has been shown that the resonant frequency shift of a thermally compensated cut increases considerably beyond the new equilibrium resonant frequency before relaxing to that final equilibrium frequency. In a similar way the time-independent temperature dependence of the velocity of surface waves in quartz as a function of the orientation of the surface and propagation direction has been determined[15]. Since the description employed is referred to a fixed reference state, the temperature-dependent skewing of the coordinate axes, which was omitted in earlier work[16,17], is automatically included and, as a consequence, the results obtained[15,18] are in considerably better agreement with experiment[16,18].

An extended version of the perturbation equations for small fields superposed on a bias, which includes the influence of the Coriolis acceleration, has been applied[19] in the calculation of the variation in the velocity of propagating surface waves with the angular velocity of the substrate. The calculation shows that for traveling surface waves there is a small sensitivity to angular velocity. The analysis also reveals that the use of a low frequency wave such as a flexural wave will significantly increase the sensitivity.

Nonlinear electroelastic equations including terms cubic in the small field variables have been obtained[20] from the aforementioned general electroelastic equations[1]. The nonlinear small field equations containing cubic terms have certain important properties that the equations containing terms no higher than quadratic in the small field variables do not. In particular, the equations containing cubic terms can consistently account for the dependence of wave velocities on wave amplitude, including surface waves, and certain nonlinear resonance phenomena, such as hysteresis, for which the equations containing terms no higher than quadratic cannot. These latter equations containing terms cubic in the small field variables have been applied in the analysis of the nonlinear problems of intermodulation[21] in thickness-shear and trapped energy resonators, as well as the closely related problem of the nonlinear vibrations[22] of such resonators. The results of this latter investigation give the dependence of the resonant frequencies on the amplitude of vibration and indicate the existence of hysteresis for large enough driving voltages. In addition, comparison with experiment shows that the nonlinear elastic constant determined from intermodulation measurements is consistent with nonlinear resonance measurements. The nonlinear differential equation describing wave propagation in anisotropic elastic rods has been obtained from the general nonlinear elastic equations, and the nonlinear rod coefficients have been expressed in terms of the fundamental linear and nonlinear anisotropic elastic constants[23]. The quadratic rod coefficients have been calculated for various orientations of quartz rods. This nonlinear purely elastic equation has been linearly coupled to the electric field and the resulting system has been applied in the analysis of intermodulation in quartz rods.

The macroscopic description of the interaction of the electromagnetic field with a finitely deformable, polarizable and magnetizable nonconducting continuum, including the spin angular momentum of the magnetization and both ionic and electronic polarization resonances, has been obtained[24]. The description does account for some of the dispersion of the effective photoelastic and dielectric coefficients as well as the dependence of the effective dielectric coefficients on the small local mechanical rotation. Moreover, the theory provides a proper description of optical activity in solids among other things. Earlier work on magnetoelasticity[25] has been extended[26] in order to permit magnetic boundary conditions at a free surface other than those of zero exchange-torque (free spins), which were the only ones possible in the previous theory.

The macroscopic description of the interaction of the electromagnetic field with an electrically semiconducting, polarizable, finitely deformable continuum has been obtained[27]. The equations have been reduced to those for the quasi-static electric field and static homogeneous magnetic field, and for the n-type semiconductor the equations in the small, but nonlinear (quadratic), field variables for small fields superposed on a static bias have been obtained from the reduced system. The linear portion of this latter more tractable system of equations has been applied to the problems of the amplification of both bulk and surface waves in piezoelectric semiconductors. The bulk wave analysis indicates the existence of a term in the amplification relation not present in previous work[28] on the subject and due to the biasing D.C. electric field and oscillating electric charge density. The surface wave analysis does not make the assumption of zero electric surface charge density employed in previous treatments[29,30] of the problem because of a semiconduction boundary condition that arises in the description developed[27]. A one-dimensional version of the general nonlinear electroelastic equations for deformable semiconductors has been employed in an analysis of the formation and propagation of acoustoelectric domains in piezoelectric semiconductors by means of the theory of acceleration waves[31]. The analysis shows that the velocity of the domain depends on the strain amplitude and that a shock tends to form in a finite time. The condition for the threshold field has been determined in the general case and when the electrical conductivity equation is reduced to the form usually employed for semiconductors the more general condition for the threshold field reduces to the known result. The same one-dimensional version of the general equations has been employed in the analysis of shock waves in piezoelectric semiconductors[32]. In the case of infinitesimal shocks the results reduce to those obtained from the acceleration wave analysis provided the piezoelectric response is linear. The integral forms of the balance laws for deformable semiconductors have been transformed from the unknown present coordinate description to the known reference coordinate description nd an improved simplified integral form for the balance of linear momentum for the semiconducting fluid has been obtained[33], which results in an improved version of the semiconduction boundary condition at a material surface of discontinuity. An interesting transformed integral form for the balance of energy has been found[33], which results in a revised energetic jump condition that is consistent with all the other jump conditions. Three-dimensional acceleration waves in piezoelectric semiconductors have been investigated[34]. One result of the investigation is a description of the formation and propagation of acoustoelectric domains in the three-dimensional case. The analysis shows that a shock tends to form in a finite time for conditions conducive to domain formation except in certain unusual cases possibly occurring with purely transverse acceleration waves. The condition for the threshold field has been determined in the general case and when the electrical conductivity equation which can be quite general in this treatment[34], is reduced to the form usually employed for semiconductors, the more general condition reduces to the anisotropic generalization of the elementary result. By removing the deformation from the description of electroelastic semiconductors[33], a fully macroscopic description of bounded semiconductors has been obtained[35], which includes the boundary conditions at the surface of the semiconductor that are required for consistency with the usual diffusion drift current equations[36]. The equations have been applied[35] to some Si-SiO_2 interfaces in the static case.

The governing nonlinear equations for two and three constituent composites have been obtained[37]. These equations should be applicable in the description of such materials as reinforced rubber. The linear equations for small dynamic fields superposed on large static fields for the two constituent composite have been obtained from the aforementioned relatively intractable nonlinear system. The linear equations have been applied in the solution of the problems of load transfer and the propagation of both bulk and surface waves in composite materials[38].

In Subject Area 2 an analysis of overtone essentially thickness-shear modes in monolithic crystal filters and trapped energy resonators has been performed[21]. More recently, an analysis of trapped energy resonators with rectangular electrodes operating in overtones of coupled thickness-shear and thickness-twist has been performed[39]. This analysis takes into consideration both dimensions of the electrode as well as the thickness of the plate and the results give excellent agreement with experiment. This latter analysis has been extended[40] to the case of two-pole monolithic crystal filters and takes all dimensions of the filter into consideration and gives excellent agreement with experiment. An analysis of a high precision contoured AT-cut quartz crystal resonator has been performed[41]. Calculations based on the theory agree extremely well with experiment. The results of these analyses have been used[12,13], along with the aforementioned perturbation theory[6] and a static treatment of thermal stresses in piezoelectric plates[7], in the determination of temperature induced frequency changes in the electroded contoured and trapped energy quartz resonators discussed earlier. An analysis of extensional modes in high coupling trapped energy resonators has been performed[42,43]. The asymptotic treatments that were employed[21,39-41] in the low coupling case are not applicable in the high coupling case. The frequency spectra obtained from the treatment reveal the existence of splittings[43], which are caused by the coupling of more than one independent set of trapped resonances. An investigation of the low frequency extensional and flexural vibrations of piezoelectric rods made of high coupling materials has been performed. In addition an investigation of the radial modes of a high coupling thin circular ferroelectric ceramic disk poled in the thickness direction has been performed[44]. The analysis includes the influence of the full anisotropy of not only crystal class C_{6v} but crystal class C_3 and its subclasses C_{3v} and C_6. Some of the results of these investigations have been incorporated in the revision of the Standards on Piezoelectric Crystals of the Institute of Electrical and Electronics Engineers, which has been published. In addition, the analysis of the radial modes of a disk has resulted in a new and more convenient method of measurement of radial mode coupling factors[44].

In Subject Area 3 the lowest symmetric and antisymmetric transmitted and reflected guided surface waves have been determined when the lowest symmetric guided surface wave in a straight guide is incident on a curved guide[45]. The radiation loss due to guide curvature was included in the calculation. One important result of this investigation is that extremely large radius-to-guide-width ratios are required in order to realize essentially nondispersive guided elastic surface wave propagation in the case of a slot in an aluminum film on a T-40 glass substrate. The surface wave dispersion curves for a slot in an aluminum-oxide film on a T-40 glass substrate have been obtained. The curves indicate that this combination of materials permits essentially nondispersive propagation for radius-to-guide-width ratios approximately half as large as those required in the case of aluminum on T-40 glass. The quasi-single-scalar function approximation technique has been used in the determination of phase velocity dispersion curves for guided surface waves for gold strips plated on lithium niobate[46]. A similar technique has been used in the analysis of the reflection of surface waves by an array of reflecting strips[47]. A calculation for a particular reflecting array[47] yields good agreement with experiment[48] and accounts for certain effects that have not been reproduced by other analyses. An investigation of transverse modal effects in periodic arrays of reflecting strips has been performed[49]. The system of approximate equations employed earlier in the straight-crested case has been extended to the variable-crested case. The reduction in the straight-crested surface wave velocity in the unplated region due to the adjacent plated regions, which is essential for the existence of the guided transverse modes, is determined by means of a piezoelectric perturbation theory[50]. Calculations performed for shorted aluminum reflecting strips on ST-cut quartz clearly reveal the existence of resonance peaks on the high frequency side of the fundamental resonance, which have been observed experimentally[48]. In addition, the calculations show that the earlier straight-crested analysis[47] overestimated the reflection coefficients somewhat. A perturbation analysis of the attenuation of surface waves due to the finite electrical conductivity of thin metal films plated on the surface and air loading has been performed[50]. Calculations have been performed for aluminum films of different thickness on ST-cut quartz. An extremely accurate relatively simple approximate solution for the fundamental antisymmetric mode of the wedge guide in isotropic materials has been obtained[51] for wedge angles larger than about 50°. Since calculations based on the analysis are relatively inexpensive, the propagation velocity as a function of wedge angle has been determined for the entire range of values of Poisson's ratio. An analysis of the propagation of surface waves in composite materials has been performed[38]. The surface wave dispersion curves for fiber reinforced composites have been determined for specific directions of surface wave propagation for a few orientations of the surface relative to the reinforcement direction. These latter results are of potential value in the nondestructive testing of fiber reinforced composites.

2. Investigation of Electromagnetism in Interaction with Conducting and Nonconducting Polarizable and Magnetizable Deformable Solid Continua

An extension[26] of an earlier, properly invariant description of saturated magnetoelastic insulators[25], which requires the magnetic boundary condition of zero exchange-torque (free spins) at a free surface has been obtained. In the magnetodynamic theory of the rigid solid, magnetic-exchange boundary conditions at a free surface other than those of zero exchange-torque are deemed to be possible[52]. Sometimes complete pinning of the spins (arbitrary resisting torque) is assumed[53] and at other times the more general condition of, say, linearly restrained spins is assumed[54]. Neither of these conditions are permitted in the existing consistent descriptions of magnetoelasticity[25,55,56] because the conditions require the existence of a mechanical resisting couple at a surface and the existing descriptions permit the existence of no such couple. However, if the existing theory is extended in such a way that nonzero surface couples[57,58], i.e., couple-tractions, can exist, magneto-exchange boundary conditions other than those of free spins at a free surface are permissible. In the purely mechanical case, Toupin[57] has shown that couple-stress theory is incomplete and should be replaced more properly by the full double-stress (or strain-gradient) theory. Consequently, in the extension the influence of the full double-stress tensor has been included. The model, which is very similar to one employed previously, consists of an electronic spin continuum coupled to a lattice continuum, with respect to which it cannot translate. Since the interacting continua have been specifically defined, the more powerful vector field rate of working procedures of Green and Rivlin[59] are employed rather than variational procedures[57]. In accordance with the above discussion, the mechanical (or lattice) continuum has been generalized in that it experiences double-force tractions in addition to the usual force traction. The two continua interact by means of a defined local magnetic field which causes equal and opposite double-force tensors to be exerted between the continua. However, it should be noted that although the lattice continuum is sensitive to entire double-force tensors, the spin continuum is sensitive only to torques. Consequently, the defined local magnetic field, which accounts for the interaction between the continua, causes the lattice continuum to experience the entire double-force tensor while the spin continuum reacts only to the torque of the equal and opposite double-force tensor. The procedures used in obtaining the governing differential equations of balance and associated constitutive equations are similar to those usually employed in continuum physics, but with the essential difference that the principle of the conservation of angular momentum of the lattice continuum, although satisfied, must be replaced by the more general and powerful principle of the invariance of the stored energy function in a rigid rotation. The two principles are equivalent in simpler cases. The conservation of angular momentum is inadequate in this more general case because the deformable mechanical continuum is sensitive to entire double-force systems rather than only to the torques of the double-force systems.

The differential equations and boundary conditions describing the behavior of an electrically polarizable, heat conducting, finitely deformable continuum in interaction with the quasi-static electric field have been derived[1] by means of a systematic application of the laws of continuum physics to a well-defined macroscopic model consisting of an electronic charge continuum coupled to a lattice continuum. The inertia of the electronic continuum is assumed to be negligible compared to the inertia of the lattice continuum. Initially, both continua occupy the same region of space and have equal and opposite charge densities so that the net charge density vanishes. In a (finite) motion the electronic continuum is permitted to displace with respect to the lattice continuum by means of an <u>infinitesimal</u> displacement field, which, by virtue of the charge densities of the two continua, accounts for the polarization. Although the two continua can displace with respect to each other, elements of each with the same initial coordinates are constrained to have equal volumes at all times and, hence, the net charge density vanishes for all times. The two continua interact by means of a defined local electric field, which causes equal and opposite forces to be exerted between the continua. When heat conduction is included the system consists of five equations in five dependent variables. When thermal considerations are eliminated, the resulting equations of nonlinear electroelasticity consist of four equations in four dependent variables. Previous variational treatments[60,61] resulted in seven equations in seven dependent variables. This system of equations is applicable in the description of the dynamic nonlinear behavior of high coupling piezoelectric materials such as lithium niobate and the linear behavior of such materials under static or quasi-static biasing fields.

The differential equations and boundary conditions quadratic in the small, but nonlinear, field variables for small fields superposed on large static biasing fields have been obtained[2] from the aforementioned general nonlinear electroelastic equations[1]. The nonlinear equations in the small field variables are considerably more tractable than the general nonlinear electroelastic equations. Application of these small field equations to polarized ferroelectrics reveals that in the linear limit the small field electroelastic equations are identical with the equations of linear piezoelectricity for the symmetry of the polarized state. The more tractable small field equations have been applied in the solution of the problem of the influence of a static biasing electric field on the thickness vibrations of a high coupling piezoelectric plate, to second order in the biasing field. To first order in the biasing field, the results indicate that the effective fifth rank tensor assumed in earlier quasi-linear work on this problem did not have correct symmetry properties because the static deformation under the biasing field was ignored. The aforementioned nonlinear equations quadratic in the small field variables have been applied to the problems of second harmonic generation and parametric excitation of surface waves in isotropic elastic solids and second harmonic generation of surface waves in anisotropic piezoelectric solids[3]. Only the formal solution is presented in the piezoelectric case because the fundamental nonlinear electroelastic coefficients have never been measured. Since the equations are quadratic, rather than cubic, in the small field variables, the solutions can be obtained consistently only to second order in the small parameter. As a consequence only the initial slopes of the harmonically generated and parametrically excited surface waves are determined. The purely elastic isotropic solid was treated as a first step, and the method of analysis there devised was applied to piezoelectric surface waves in anisotropic solids. The quadratic nonlinear small field equations can be used in connection with an experimental program to completely determine the fundamental tensorial material coefficients governing second harmonic generation and parametric excitation in high coupling materials such as lithium niobate. Once the fundamental tensorial parameters have been evaluated, the aforementioned solution of the problem of second harmonic generation of surface waves in anisotropic piezoelectric solids can be used to determine the practical orientations of the materials, if indeed there be any, for surface wave parametric interactions or any other interactions of interest.

The linear limit of the equations for small fields superposed on a bias has been applied in a direct calculation of the influence of a flexural biasing state on the velocity of surface waves in piezoelectric solids[4]. The influence of the biasing stresses appears in the boundary conditions as well as the differential equations. Calculations have been made for Y-Z lithium niobate and ST-cut quartz. The results of this investigation have been useful in the interpretation of measurements on pressure sensing acoustic surface wave resonators[5]. A perturbation formulation of the same linear equations for small fields superposed on a bias has been obtained[6]. The bias may be either electrical or mechanical or both. This formulation provides a relatively simple extremely accurate description of the influence of biasing electric fields, stresses and strains on such things as the velocity of surface waves and the resonant frequencies of vibrating piezoelectric structures, once the bias is known. The perturbation formulation has been applied[4] in the determination of the aforementioned influence of a flexural biasing state on the velocity of piezoelectric surface waves, and the calculations reveal that the results are every bit as accurate as those obtained from the direct calculation. In fact, since the perturbation procedure can readily treat spatially varying biasing states, for which a direct calculation cannot be performed, it has significant advantages over a direct calculation for this type of purpose. The results of the perturbation calculation show[4] that for substrate thickness-to-wavelength ratios well within the practical range, the spatial variation of the biasing state has an appreciable influence on the velocity of surface waves. In addition, the perturbation formulation has been applied in the determination of temperature induced frequency changes in electroded doubly-rotated quartz plates vibrating in thickness-modes[7]. In order to calculate the actual frequency change for doubly-rotated quartz plates, the first temperature derivatives of the fundamental elastic constants of quartz had to be determined[8] from the original data[9] from which the temperature derivatives of the effective constants were obtained[9]. The newly defined temperature derivatives are required because the existing temperature derivatives of the effective constants cannot conveniently be employed in calculations of thermally induced frequency changes in electroded quartz resonators. This is essentially a result of the fact that a knowledge of temperature induced biasing strains and the third order elastic constants cannot be used in the existing linearly based formulation, but requires a proper nonlinearly based formalism. The biasing state is determined from a system of approximate plate equations for the determination of thermal stresses in piezoelectric plates with thin films, which are large compared with the thickness of the piezoelectric plate, plated on the surfaces[7]. Doubly-rotated quartz plates with identical rectangular electrodes on the major surfaces have been considered in detail. The thermally induced biasing deformation and the aforementioned first temperature derivatives of the fundamental elastic constants of quartz[8] are used in the equation for the perturbation in eigenfrequency[6] to find the frequency change with temperature of electroded quartz plates[7]. Calculations have been performed for doubly-rotated cuts of quartz vibrating in thickness-modes.

The system of approximate plate equations for the determination of thermal stresses in electroded piezoelectric plates has also been applied to both contoured AT-cut quartz crystal resonators[12] and AT-cut quartz trapped energy resonators[13]. In both instances the changes in resonant frequency resulting from the thermally induced biasing stresses and strains and the first temperature derivatives of the fundamental elastic constants of quartz[8] were determined from the equation for the perturbation in eigenfrequency[6] due to a bias. In the case of the contoured resonator[12] the changes in resonant frequency with temperature were calculated for the fundamental and some of the anharmonic overtones of the fundamental and some of the harmonic overtones which were obtained in an analysis of overtone modes in contoured crystal resonators[10]. In the case of the trapped energy resonator[13] the changes in resonant frequency with temperature were calculated for the fundamental and a number of harmonic and anharmonic overtone trapped energy modes for rectangular electrodes oriented in various directions with respect to the digonal axis on AT-cut quartz plates. In this way the dependence of the change in resonant frequency per $^{\circ}$K on both the orientation of the rectangular electrodes and the electrode geometry was determined. In addition, the change in frequency with temperature of the fundamental trapped energy mode for rectangular electrodes oriented along the digonal axis of the AT-cut quartz plate has been obtained as a function of the electrode geometry and plate thickness. The results obtained in these investigations[12,13] are related to the dependence of the apparent shift in angle of the zero temperature cut on either the contouring[12] or electrode geometry[13].

When a quartz resonator is subject to a change in the ambient temperature it undergoes a temporal nonuniform temperature distribution which causes the resonant frequency to drift with time[62,63], until the new thermal equilibrium state is reached. In a treatment[14] of this problem the time-dependent temperature distribution in the quartz plate was obtained from the uncoupled heat conduction equation subject to appropriate initial and boundary conditions. The time-dependent thermally induced biasing state was determined from the exact equations of static linear thermoelasticity for the unelectroded plate and from the aforementioned system of approximate thermoelastic extensional plate equations for the electroded plate. The time-dependent change in resonant frequency for doubly-rotated quartz thickness-mode resonators resulting from the thermally induced biasing state was determined from the equation for the perturbation in eigenfrequency due to a bias as in the time-independent cases. Among other things the analysis reveals that in a thermally uncompensated cut the influence of the electrodes on the change in frequency is small and the frequency increases monotonically from one equilibrium state to the other. On the other hand, since in a thermally compensated cut the change in frequency in the absence of the electrodes is essentially negligible, the thermally induced biasing deformation state resulting from the presence of the electrodes is the dominant factor causing the frequency change. As a consequence, the calculated frequency shift increases in time considerably beyond the new equilibrium resonant frequency, in conformity with experimental data[62,63]. Calculations have been performed for a number of thermally compensated as well as uncompensated cuts of quartz for some physically meaningful temperature inputs.

Nonlinear electroelastic equations including terms cubic in the small field variables have been obtained[20] from the aforementioned general electroelastic equations[1]. The nonlinear small field equations containing cubic terms have certain important properties that the equations containing terms no higher than quadratic in the small field variables do not. In particular, the equations containing cubic terms can consistently account for the dependence of wave velocities on wave amplitude, including surface waves, and certain nonlinear resonance phenomena, such as hysteresis, for which the equations quadratic in the small field variables cannot. It has been shown[64] that although progressively higher order nonlinear theories are required to consistently describe the generation of successively higher harmonics, only the quadratic nonlinear term is necessary to account for the measurement of all longitudinal harmonics in harmonic generation experiments because of the order of magnitudes of successively increasing order elastic constants and the number of wavelengths over which harmonic generation measurements are usually made. The equations containing cubic terms have been applied in the analysis of intermodulation[21] and nonlinear vibrations[22] in thickness-shear and trapped energy resonators. In each instance lumped parameter representations of the solutions, which are valid in the vicinity of a resonance, were obtained and the crystal was incorporated in a circuit. In the intermodulation cases the relation between the intermodulation and driving voltages was obtained. In the nonlinear resonance cases current response curves as a function of frequency for various driving voltages were obtained and the dependence of resonant frequency on the amplitude of the current was determined. The intermodulation theory compared favorably with experiments using AT-cut quartz resonators operating in the fundamental and third overtone essentially thickness-shear modes and enabled an estimate of the fourth order elastic constant $\underset{\sim}{c}^{E}_{6666}$ for AT-cut quartz to be made. A comparison of the results of the nonlinear resonance calculations with experiment reveals that the value of the fourth order elastic constant determined from the intermodulation measurements is consistent with nonlinear resonance measurements.

The one-dimensional scalar nonlinear differential equation describing the extensional motion of a thin piezoelectric rod oriented in an arbitrary direction with respect to the principal axes of the crystal has been obtained[23] from the general nonlinear three-dimensional equations of electroelasticity. Only the elastic nonlinearities are included in the description. The electrical behavior is taken to be linear since quartz has small piezoelectric coupling. The treatment provides the relation between the quadratic and cubic nonlinear extensional coefficients of the rod and the fundamental anisotropic constants of second, third and fourth order along with the well-known relation between Young's modulus and the fundamental second order elastic constants. The quadratic rod coefficients have been calculated for various orientations of quartz rods with respect to the principal axes of the crystal. Such calculations cannot be performed for the cubic rod coefficients because the fourth order elastic constants of quartz, on which the cubic coefficients depend, are not presently known. The nonlinear extensional equation has been applied in the analysis of intermodulation in quartz rods. A lumped parameter representation of the solution, which is valid in the vicinity of a resonance, was obtained and the influence of the external circuitry was included in the treatment.

The differential equations and boundary conditions describing the behavior of a finitely deformable, polarizable and magnetizable, heat conducting but electrically nonconducting continuum in interaction with the electromagnetic field have been derived[24]. Magnetic spin resonance and both ionic and electronic polarization resonances have been included in the analysis. In essence the resulting equations couple previous work in magnetoelasticity[25,55] with an extended version of the recent work in electroelasticity[1]. The description is not Lorentz invariant, but that is not a severe limitation because the material velocities encountered in practice are considerably less than the speed of light. However, it is to be noted that the description should be accurate to terms linear in the ratio of the material velocity to the speed of light and, thus, should be capable of accurately describing very small velocity effects. Moreover, it may ultimately be possible to make the description Lorentz invariant. The model, from which the description is obtained, consists of an electronic charge and spin continuum coupled to a lattice (mechanical) continuum. The lattice continuum is somewhat more complicated than any considered heretofore, in that it consists of two interpenetrating ionic continua which can displace with respect to each other and, thus, produce ionic polarization. Electronic spin resonance and both electronic and ionic polarization resonances have been incorporated in the description. The identified continua interact by means of defined local electric and magnetic material fields, which cause balancing forces and couples to be exerted between the continua. In this instance, Maxwell's full electromagnetic equations are required in place of the equations of either electrostatics or magnetostatics, which are all that were required heretofore. The resulting system of equations reduces to fifteen equations in fifteen dependent variables when heat conduction is excluded. The resulting equations differ in certain significant respects from those of Toupin[65] on the electrodynamics of finitely deformable, polarizable continua. In particular, Toupin did not include magnetization in his treatment. In addition, he did not attempt to define a macroscopic model in anywhere near as much detail as our model, which includes magnetization and can distinguish between ionic and electronic polarization and inertia and, hence, includes both polarization resonances. Since the thermodynamic state function depends on the ionic polarization gradient, the theory provides a proper description of optical activity in solids[66], among other things. In addition, the equations show that the description accounts for some of the dispersion of the effective photoelastic and dielectric coefficients as well as the dependence of the spatially varying effective dielectric coefficients on the small local mechanical rotation. Earlier treatments[67] of such scattering have included the effect of the small strain only because the starting equations were not rotationally invariant. A treatment by Nelson and Lax[68] has appeared, which includes the effect of the small local mechanical rotation as well as the small strain, but not the dispersion.

The differential equations and boundary conditions describing the interaction of the electromagnetic field with an electrically semiconducting, polarizable, finitely deformable, heat conducting continuum have been obtained[27]. The resulting equations have been reduced to those for the quasi-static electric field and static homogeneous magnetic field. In the absence of heat conduction, for the n-type semiconductor, nonlinear equations quadratic in the small field variables for small fields superposed on a D.C. bias have been obtained from the reduced system. The linear portion of this latter, more tractable, system of equations has been applied to the problems of the amplification of both bulk and surface waves in piezoelectric semiconductors. Among other things, the bulk wave analysis indicates the existence of a term in the amplification relation not present in previous work on the subject[28] and due to a mechanical body force caused by the D.C. electric field and oscillating electric charge density. In addition the surface wave analysis does not make the assumption of zero electric surface charge density employed in previous formulations[29,30] of the problem because of the existence of a semiconduction boundary condition that arises naturally in the description. The surface charge density accompanying a surface wave can then be calculated a posteriori. Furthermore, previous work[28-30] on wave propagation in piezoelectric semiconductors subject to a biasing D.C. electric field does not exhibit any dependence of the effective material constants on the biasing D.C. field, as does this treatment.

A one-dimensional version of the general nonlinear electroelastic equations for deformable n-type semiconductors has been employed in the analysis of the formation and propagation of acoustoelectric domains in piezoelectric semiconductors by means of the theory of acceleration waves[31]. The mechanical and dielectric nonlinearities are automatically included in the analysis as well as the semiconduction nonlinearity. The analysis indicates that the velocity of the domain depends on the strain amplitude and that a shock tends to form in a finite time for conditions conducive to domain formation. The condition for the threshold field has been determined under rather general circumstances and when the electrical conductivity equation, which can be quite general in the treatment, is specialized to the form usually employed for semiconductors, the more general condition for the threshold field reduces to the known result. The same one-dimensional version of the general equations has been employed in the analysis of shock waves in piezoelectric semiconductors[32]. Some physical information concerning the evolutionary behavior of certain simple types of shock have been obtained from the rather complicated shock amplitude equation. In the case of infinitesimal shocks the results reduce to those obtained from the one-dimensional acceleration wave analysis provided the piezoelectric response is linear.

The integral forms of the balance laws for deformable semiconductors have been transformed from the unknown present coordinate description to the known reference coordinate description[33], which is the form needed in the treatment of problems. An improved simplified integral form for the balance of linear momentum for the semiconducting fluid is obtained, which results in an improved version of the semiconduction boundary condition at a material surface of discontinuity. In addition, the previous existing integral form of the equation of the balance of energy is transformed to a different form, which is equivalent to the original form only when the field variables are differentiable. An energetic jump condition across a moving nonmaterial surface of discontinuity has been obtained from the revised integral form. The revised energetic jump condition is consistent with all the other jump conditions obtained from the other integral forms.

An analysis of three-dimensional acceleration waves in piezoelectric semiconductors has been performed[34]. While the previous one-dimensional analysis[31] was restricted to longitudinal acceleration waves and domains of longitudinal strain only, the present three-dimensional analysis treats waves with arbitrary three-dimensional deformation fields with wave surfaces of arbitrary shape, and thereby enables the consideration of acoustoelectric domains of arbitrary shape with arbitrary deformation fields. In particular, this three-dimensional analysis is necessary for the treatment of the important special case of domains with transverse deformation fields and plane surfaces normal to the propagation direction because one-dimensional treatments are always restricted to longitudinal waves and, hence, longitudinal acoustoelectric domains. The analysis indicates that a shock tends to form in a finite time for conditions conducive to domain formation except in certain unusual cases possibly occurring with purely transverse acceleration waves. The condition for the threshold field has been determined under rather general circumstances and when the electrical conduction equation, which can be quite general in this treatment, is specialized to the form usually employed for anisotropic semiconductors, the more general condition reduces to the anisotropic generalization of the well-known elementary result.

By eliminating the deformation from the description of electroelastic semiconductors, a fully macroscopic description of semiconductors has been found[35], which includes the boundary conditions at the surface of the semiconductor that are required for consistency with the usual diffusion-drift current equations[36]. As in all field theories, e.g., electromagnetism, both the boundary conditions and the differential equations are obtained from the same governing integral forms. The new boundary conditions relate the jump discontinuities in the chemical potentials across the interface to the forces exerted by the lattice on the charge carriers which prevent the carriers from leaving the solid. The expressions for the forces in the static case have been found and the values of the material surface coefficients appearing therein were obtained from quasi-static MOS C-V measurements for some particular Si-SiO_2 interfaces.

The nonlinear differential equations and boundary conditions for finitely deformable, heat conducting, N-constituent composites have been obtained[37]. These equations should be applicable in the description of such materials as reinforced rubber. The influence of viscous dissipation is included in the general treatment. Although the motion of the combined composite continuum may be arbitrarily large, the relative displacement of the individual constituents is required to be infinitesimal in order that the solid composite not rupture. The linear version of the equations in the absence of heat conduction and viscosity have been obtained for the two-constituent composite. The linear equations have been exhibited in detail for the isotropic and transversely isotropic material symmetries. Plane wave solutions in the isotropic case reveal the existence of high frequency (optical type) as well as low frequency (acoustic type) branches, and all waves are dispersive. The linear equations in the transversely isotropic case have been applied in the solution of problems of load transfer and surface wave propagation in composite materials[38]. The surface wave problems and solutions along with certain numerical results obtained are discussed in Section 4.

3. Investigation of the Vibrations of Piezoelectric Plates and Rods

An analysis of a partially plated piezoelectric plate oscillating in the vicinity of the fundamental thickness-shear frequency and including the influence of the electric fields outside the unelectroded portions of the plate, has been performed[69]. The eigensolution shows that when the electric fields outside the unelectroded portion of the plate are taken into account, the previously elastically determined purely imaginary dispersion curves become complex. An important consequence of the existence of complex branches, as opposed to purely imaginary branches, is that no significant amount of quasi-static electric field energy is radiated from an unelectroded surface of a piezoelectric resonator in the steady-state and, hence, electromagnetic radiation does not contribute significantly to the quality factor (Q) of the vibrator. Another interesting fact exhibited by the piezoelectric eigensolution is that energy trapping disappears at a frequency slightly below the thickness-shear frequency of the unelectroded section of the plate. It should be noted that the existence of complex - rather than purely imaginary - branches in this type of piezoelectric situation, and the consequent lack of a significant amount of electromagnetic energy radiation, is a reasonably general result, which should hold in almost all other similar situations.

The relation of electromechanical coupling factors to the fundamental material constants for thickness vibrating piezoelectric plates has been determined[70]. It has been shown that when thickness resonator and transducer materials possess the appropriate symmetry, it is meaningful to discuss and measure electromechanical coupling coefficients. However, when materials do not possess the appropriate symmetry, the coupling factor approach is not meaningful. In measuring such materials one must proceed by first making the requisite measurements to find the fundamental material constants and then analytically finding the practical orientations, for which the elementary equivalent circuit transducer analysis holds[71].

An analysis of the radial modes of a high coupling thin circular piezoelectric disk has been performed for the full anisotropy of crystal class C_3 with the 3-fold axis normal to the circular surfaces of the disk. The analysis naturally holds for the subclasses C_{3v}, C_6 and C_{6v}, which is the crystal class for the polarized ferroelectric ceramic. This analysis has resulted in a new and more convenient method of measurement of radial mode coupling factors[44].

An analysis of overtone modes in trapped energy resonators vibrating in essentially thickness-shear has been performed[21]. Closed form asymptotic expressions for the dispersion curves of the overtone thickness-shear branches in the vicinity of the thickness-shear point have been obtained from the appropriate solutions of the three-dimensional linear piezoelectric equations for both the unelectroded and fully electroded plate. Approximate edge conditions to be satisfied at a junction between an electroded and unelectroded region were determined in a manner exhibiting the natural limitations inherent in the approximation. The dispersion relations and edge conditions have been applied in the analysis of the steady-state vibrations of a trapped energy resonator, and a lumped parameter representation of the admittance, which is valid in the vicinity of a resonance, was obtained.

An analysis of trapped energy resonators with rectangular electrodes operating in overtones of coupled thickness-shear and thickness-twist has been performed[39]. Closed form asymptotic expressions for the frequency wave-number dispersion relations for the fundamental and odd overtone coupled thickness-shear and thickness-twist waves in the vicinity of the thickness-shear point have been obtained for both the unelectroded and fully electroded plate. Approximate boundary conditions at junctions between electroded and unelectroded regions were determined in a manner exhibiting the natural limitations inherent in the approximation. The boundary conditions to be satisfied at the junctions between the corner region and the adjacent regions were obtained from an extended version of the variational principle[72] of linear piezoelectricity. The dispersion relations and edge conditions have been applied in the analysis of the steady-state vibrations of a trapped energy resonator, and a lumped parameter representation of the admittance, which is valid in the vicinity of a resonance, was obtained.

The above-mentioned treatment of overtone modes in trapped energy resonators has been extended to the case of two-pole monolithic crystal filters[40]. The asymptotic dispersion relations for the fundamental and odd overtone coupled thickness-shear and thickness-twist waves near cutoff along with the simple approximate edge conditions at junctions have been applied in the analysis of the steady-state vibrations of two-pole monolithic crystal filters, and a lumped parameter representation of the admittance matrix for the two port device has been obtained.

An analysis of contoured quartz crystal resonators has been performed[41]. The analysis holds for the fundamental and harmonic overtones of thickness-shear, as well as all the anharmonic overtones of each thickness-shear mode. In this work the analysis of trapped energy resonators operating in overtones of coupled thickness-shear and thickness-twist vibrations is extended to the case of plates with slowly varying thickness. A lumped parameter representation of the admittance, which is valid in the vicinity of a resonance, has been obtained. No adjustable parameters are required in the theory. Calculations based on the theory agree extremely well with measurements on contoured crystal resonators.

An analysis of extensional modes in high coupling trapped energy resonators has been performed[42]. Although the fundamental thickness-shear mode, which is used most often, may always be trapped by a set of connected electrodes on a flat piezoelectric plate, the fundamental thickness-extensional mode usually may not be trapped in the same manner. However, this latter mode may be trapped in the foregoing manner on a flat plate of high coupling material, such as PZT-7A, for which the existence of trapping has already been verified experimentally[73]. The simplified asymptotic treatments[39] that have been employed in the low coupling case are not applicable in the high coupling case, which requires a more complete analysis because of the large frequency range in which trapping occurs. Consequently, the dispersion curves have been obtained for the pertinent fundamental extensional waves in an infinite PZT-7A plate for both the unelectroded case and that of shorted electrodes. Combinations of these solutions have been employed in an appropriate variational principle[72] of linear piezoelectricity in order to satisfy the remaining boundary conditions at the junction between the electroded and unelectroded regions approximately and obtain the frequency spectra for the thickness-extensional trapped energy modes in a PZT-7A plate with strip electrodes. In addition to considering the dominant and most important wave in each region in the calculation of the trapped mode, the influence of other pertinent plate waves have been included in an extended calculation[43].

Although all pertinent waves in the electroded region are retained in the calculation, only those that decay with distance and do not radiate energy in an infinite plate are included in the unelectroded region. The frequency spectra obtained from the extended calculation are considerably more complicated than those obtained from the first simplified treatment in that splittings appear, which are caused by the coupling of more than one independent set of trapped resonances. Although the fundamental trapped mode usually exhibits the ordinary shape with a single maximum at the center of the electrode, in the vicinity of a splitting it exhibits a more complicated shape with an even number of maxima across the electrode and a local minimum at the center. At certain discrete operating frequencies this latter mode shape has been observed experimentally.

4. Investigation of Elastic Surface Waves Guided by Thin Films

The lowest symmetric and antisymmetric transmitted and reflected guided waves have been calculated when the lowest symmetric guided wave in a straight guide is incident on a circular guide[45]. The analysis is based on an extension of the planar scalar wave approximation, previously used for straight guides[74] to curved guides with large radius-to-guide-width ratios. The analysis shows that some radiation is always present in a curved guide. Approximate bound modes were calculated for the circular guide and used to approximately evaluate the radiation attenuation for each mode of the circular guide. The approximation procedures are extremely accurate when the radiation attenuation is small, but they become highly inaccurate as the radiation attenuation becomes large. This means that the procedures employed are good in practical instances but no good in impractical ones. This loss of accuracy results in the definition of a critical radius-to-guide-width ratio, beyond which there is negligible guidance. In fact, this limit can never be realized in practice because the accompanying radiation would be prohibitively large. The transmitted and reflected waves, for the case of a straight guide incident on a quarter circle guide which exits into another straight guide, are obtained by retaining only the two lowest modes in each region and using the orthogonality of the modes. The radiation attenuation of each mode in the circular guide is included in the calculation. The calculations were performed for the cases of gold strips on fused-silica substrates and slots in aluminum films on T-40 glass substrates. For the case of gold strips on fused-silica substrates the results of the radiation attenuation calculation were subsequently verified experimentally[75]. One important result of the calculation is that radius-to-guide-width ratios larger than 560 are required in order to realize essentially nondispersive guided elastic surface wave propagation in the case of a slot in an aluminum film on a T-40 glass substrate.

The above-mentioned analysis of the circular slot guide results in a simple method of determining the permissible radius-to-guide-width ratios for guided surface wave propagation in the essentially nondispersive region. The surface wave dispersion curves for a slot in an aluminum oxide film on a T-40 glass substrate have been obtained. Application of the aforementioned simple method to these dispersion curves reveals that radius-to-guide-width ratios of 270 are permissible for essentially nondispersive propagation for this combination of materials.

The problem of guided surface wave propagation in straight guides on anisotropic substrates has been treated by means of a quasi-single-scalar function approximation technique[46]. The guided surface wave may be either elastic or piezoelectric. The differential equations and edge conditions for the quasi-single-scalar function approximation technique have been obtained by means of a suitably selected expansion in the variational principle of linear piezoelectricity and an integration with respect to depth. The input expansions are determined from the straight-crested surface wave solutions of the three-dimensional equations of linear piezoelectricity for both the plated and unplated substrates. Application of the equations to the isotropic cases treated earlier[74] by an ad hoc approximation technique, which has been shown to be accurate by comparison with experiment, reveals that the derived equations predict dispersion curves identical with those obtained from the ad hoc treatment. However, the present equations are applicable in the anisotropic case including the piezoelectric case, and for forced wave problems. The phase velocity dispersion curves for straight-crested waves for a thin gold film plated on Y-X and Y-Z lithium niobate have been obtained. The analysis includes the influence of electrode stiffness, inertia and shorting. These straight-crested solutions and associated dispersion curves have been used along with the straight-crested solution for unplated lithium niobate to obtain the approximate solution for gold strips plated on Y-X and Y-Z lithium niobate from the quasi-single-scalar function approximate procedure. The dispersion curves for gold strips plated on Y-X and Y-Z lithium niobate have been determined for a number of width-to-thickness ratios. Although the dispersion curves for the guided Y-Z surface waves turn out as expected, the dispersion curves for the guided Y-X surface waves have a very interesting and unusual shape with a trough and a peak, which results in two wavelength ranges of essentially nondispersive propagation. The unusual shape is caused by mode coupling resulting from the fact that the dispersion curves for two straight-crested surface waves intersect in the propagation range of interest. The two waves can be identified as the extended Rayleigh wave and the fundamental quasi-Love wave, respectively.

A quasi-scalar function approximation technique[47] has been devised to treat the problem of the reflection of surface waves by an array of reflecting strips plated on a substrate. The differential equations and edge conditions have been obtained from Hamilton's principle for linear piezoelectric media by assuming suitable depth behavior and integrating with respect to depth. The assumed behavior with depth was determined from the known straight-crested solutions of the three-dimensional equations for both the plated and unplated substrate. The approximate equations are expressed in terms of the known fundamental material constants and no measurement of model parameters is required. The approximate equations, which admit of a transmission line representation, have been applied in the analysis of surface wave reflection by both uniformly and nonuniformly spaced arrays of reflecting strips on various substrates. A calculation for a particular reflecting array on Y-Z lithium niobate, which has been built and measured, yields good agreement with experiment[48]. The calculated reflection curves indicate a slight asymmetry for heavier film materials on account of the dispersion caused by the strips. Although this effect has been observed experimentally, it has not been reproduced by other analytical models.

A perturbation analysis of the attenuation and dispersion of surface waves has been performed[50]. The attenuation due to both the finite electrical conductivity of thin metal films plated on the surface and air loading has been determined along with the dispersion due to thin films plated on the surface. The influence of the viscosity of the air is included in the treatment. An approximate thin plate conductivity equation has been obtained, which enables the entire electrical conductivity effect of the plating to be treated as a boundary condition at the surface of the substrate, thereby simplifying the analysis considerably. Numerical results have been obtained for aluminum films of varying thickness on ST-cut quartz and Y-Z lithium niobate. The numerical results reveal that the inclusion of viscosity in the air loading analysis is essential for the calculated attenuation to be in excellent agreement with experiment.

An investigation of transverse modal effects in periodic arrays of reflecting strips has been performed[49]. The system of approximate surface wave equations obtained earlier[47] from a variational principle and employed in the treatment of straight-crested surface waves has been extended to the case of variable-crested surface waves. However, in the present case a reduction in the straight-crested surface wave velocity in the unplated region due to the adjacent plated regions, which is essential for the existence of guided transverse modes, is determined by means of a perturbation procedure. At the same time an increase in the straight-crested surface wave velocity in the plated region due to the adjacent unplated regions is found. The extended system of equations, which admits of a parallel transmission line representation, has been applied in the analysis of the reflection of variable-crested surface waves by uniformly spaced arrays. The response to a rectangular input of a particular reflecting array consisting of shorted aluminum strips on ST-cut quartz has been calculated. The calculations clearly reveal the existence of small resonance peaks (spurs) on the high frequency side of the fundamental resonance which have been observed experimentally[48]. In addition, the results of the present analysis reveal that the earlier straight-crested analysis[47] overestimated the reflection coefficient somewhat for a given number of reflecting strips.

An extremely accurate approximate solution for the fundamental non-dispersive antisymmetric mode of the wedge guide in isotropic materials for wedge angles ranging from about 50° up to the angle at which the phase velocity of the mode equals the surface wave velocity of the material and guidance disappears, which is beyond 90°, has been obtained[51]. The solution consists of only two variable-crested surface wave terms, and since calculations based on the analysis are relatively inexpensive, the propagation velocity as a function of wedge angle has been obtained for the entire range of values of Poisson's ratio.

The temperature dependence of the velocity of surface waves in quartz has been calculated[15] as a function of the orientation of the surface using the newly defined first temperature derivatives of the fundamental elastic constants of quartz[8]. Previous work[16,17] on the temperature dependence of the velocity of surface waves in quartz employed the temperature derivatives of the effective constants[9], which are referred to the variable temperature dependent intermediate position rather than the fixed reference position. Not only does the intermediate coordinate system extend under a temperature increase, but it shears or skews as well. Although in the existing treatments[16,17] of the temperature dependence of surface wave velocity the change in density was properly included, the associated skewing of the coordinate axes was omitted. Since the description we employ is referred to a fixed reference state, the mass density and geometry do not change and the temperature coefficient of natural velocity, which we determine, is the negative of the temperature coefficient of delay. The results obtained[15,18] are in considerably better agreement with experiment[16,18] and a new zero temperature cut and propagation direction (called the SST-cut) for surface waves in quartz has been determined[76], which has higher piezoelectric coupling and lower loss than the commonly used ST-cut.

In inertial guidance the movement of a vehicle is directed from one point to another using measurements from sensing devices. These measurements do not entail the determination of fields outside the vehicle[77]. Two types of sensing devices are used; accelerometers and gyroscopes. The possibility of implementing both of these devices using surface waves in an essentially monolithic fashion has been investigated[19]. If three surface wave resonators such as those used in the surface wave pressure transducer[5] are loaded by means of relatively large masses, which when subject to acceleration can impart large biasing loads to the substrate and thereby cause changes in the resonant frequency, the structure is an accelerometer. The function of the gyroscope can be simulated if rotation rate can be sensed. It is well known that if a point moves with a velocity relative to a rotating substrate the moving point experiences the Coriolis acceleration[78], which is linear in and normal to both the angular velocity of the substrate and relative velocity of the moving point. Consequently, the velocity of propagation of a surface wave will vary with the angular velocity of the substrate. Thus, by monitoring the velocity of three distinct surface waves the angular velocity can in principle be determined. Calculations have been performed[19] for a particular surface wave accelerometer structure loaded in flexure using a well established perturbation procedure[6] and compared with measurement. Additional calculations were performed[19] for the surface wave rotation rate sensor using an extension[19] of the perturbation procedure that includes the Coriolis effect. The magnitude of the effect has been determined for surface waves on ST-cut quartz and it was shown that the sensitivity is relatively small, but that it can be substantially improved by using a lower frequency mode. It has also been shown that although the effect exists for traveling waves, it does not exist for standing waves such as occur in the surface wave resonator.

The linear equations for a two-constituent composite discussed in Section 2 have been applied in the analysis of surface waves propagating in the direction of the fiber reinforcement[38]. Since the defined constants occurring in the linear equations for a two-constituent composite material have never been measured, calculations cannot be performed. When the model is simplified sufficiently, the effective constants in the description can be partially estimated from the known elastic constants of the individual constituents in the composite. With the reduced equations calculations have been performed for surface waves propagating both in and normal to the direction of the fiber reinforcement. The calculations indicate the existence of a high (optical type) as well as a low (acoustic type) surface wave mode, both of which are dispersive. It is felt that the optical type mode is an analytical consequence of the simplified model and does not actually exist. The dispersion of the acoustic type surface wave mode could provide a means of nondestructively evaluating the integrity of a fiber reinforced composite material.

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